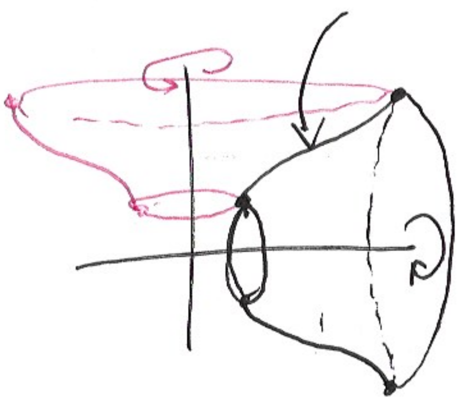
ARCLENGTH

$$s=\int_{t_a}^{t_b}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

IF $\frac{dx}{dt}, \frac{dy}{dt}$ ARE NEVER BOTH 0 SIMULTANEOUSLY

$$ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

SURFACE AREA OF REVOLUTION



$\int 2\pi y\, ds$ IF REVOLVE AROUND X-AXIS

$\int 2\pi x\, ds$ y-

$$\int 2\pi y\, ds=\int 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

eg. THE CURVE $x = 2t^2 + \frac{1}{t}$, $y = 8\sqrt{t}$, $t \in [1,4]$ IS REVOLVED AROUND THE X-AXIS

FIND THE RESULTING S.A.

$$\int 2\pi y\, ds = \int_1^4 2\pi \cdot 8\sqrt{t}\sqrt{(4t - t^{-2})^2 + (4t^{-\frac{1}{2}})^2}\, dt$$

$$= 16\pi \int_1^4 t^{\frac{1}{2}} \sqrt{(16t^2 - 8t^{-1} + t^{-4}) + (16t^{-1})}\, dt$$

$$= 16\pi \int_1^4 t^{\frac{1}{2}} \sqrt{16t^2 + 8t^{-1} + t^{-4}}\, dt$$

$$= 16\pi \int_1^4 t^{\frac{1}{2}} \sqrt{(4t + t^{-2})^2}\, dt$$

$$= 16\pi \int_1^4 t^{\frac{1}{2}} |4t + t^{-2}|\, dt$$

$$= 16\pi \int_1^4 t^{\frac{1}{2}} (4t + t^{-2})\, dt$$

$$= 16\pi \int_1^4 (4t^{\frac{3}{2}} + t^{-\frac{3}{2}})\, dt$$

$$= 16\pi \left(4 \cdot \tfrac{2}{5} t^{\frac{5}{2}} - 2 \cdot t^{-\frac{1}{2}}\right)\Big|_1^4$$

$$= 16\pi \left(\tfrac{8}{5}(4^{\frac{5}{2}} - 1) - 2(4^{-\frac{1}{2}} - 1)\right)$$

$$= 16\pi \left(\tfrac{8}{5} \cdot 31 - 2 \cdot (-\tfrac{1}{2})\right)$$

$$= 16\pi \left(\tfrac{248}{5} + 1\right)$$

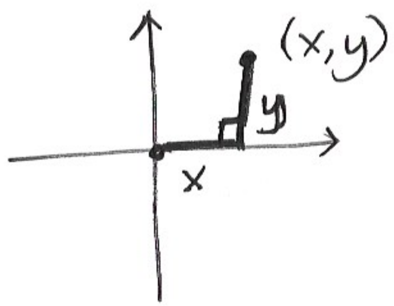
$$= 16\pi \cdot \tfrac{253}{5}$$

$$= \frac{16 \cdot 253}{5}\pi$$

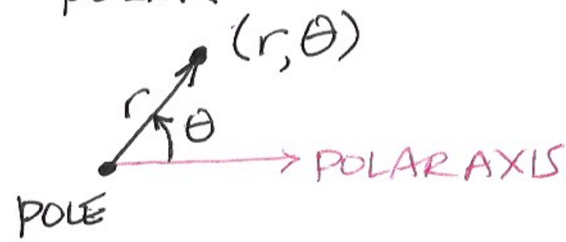
$$4^{\frac{5}{2}} = (4^{\frac{1}{2}})^5 = 2^5 = 32$$

10.3 POLAR

RECTANGULAR



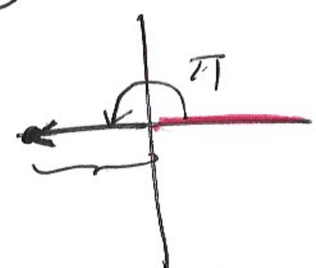
POLAR



$(r,\theta) = (5, \frac{3\pi}{2})$

$(x,y) = (0,-5)$

$(x,y) = (-8,0)$



$(r,\theta) = (8,\pi)$

OR $(8, k\pi)$ k IS ODD

(r,θ) REFERS TO THE SAME POINT AS $(r, \theta + 2k\pi)$ FOR ALL $k \in \mathbb{Z}$

← OR ANY ANGLE CO-TERMINAL WITH π

IN TERMS OF x, y, r

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$y = r\sin\theta \qquad x = r\cos\theta$$

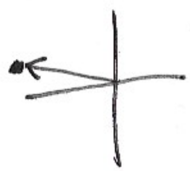
TO CONVERT (r,θ) TO (x,y)

$$(x,y) = (r\cos\theta, r\sin\theta)$$

$(r,\theta) = (4, \frac{5\pi}{6})$ CORRESPONDS TO

$$(x,y) = \left(4\cos\frac{5\pi}{6}, 4\sin\frac{5\pi}{6}\right)$$
$$= \left(4\left(-\frac{\sqrt{3}}{2}\right), 4\left(\frac{1}{2}\right)\right)$$
$$= (-2\sqrt{3}, 2)$$

$\downarrow$



SANITY CHECK:

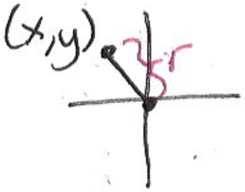
SIGNS ✓

SIZE ($|x| > |y|$ → MORE SIDEWAYS THAN UP/DOWN) ✓

TO CONVERT (x,y) TO (r,θ)

$x = r\cos\theta$

$y = r\sin\theta$



$r = \sqrt{x^2+y^2}$ $\quad r^2 = x^2+y^2$

$\cancel{\theta = \underbrace{\tan^{-1}\frac{y}{x}}_{\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}}$ $\quad \tan\theta = \frac{y}{x}$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$